Hydrogen in Electrodynamics. VIII
The Half-Integer Spin

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After a discussion of the one-component Schrödinger (1926) and the four-component Dirac (1928) representation of hydrogen it is shown that the six-component electrodyamic picture turns out to be considerably simpler and clearer. The computational effort is reduced to a fraction.

1. Historical Comment

Going through the familiar hydrogen models, where the one by J. J. Thomson may well be considered as the first one, we see that the concept of spin has not established itself right from the beginning.

Thomson's hydrogen model of 1904 [1] contains an electron within a spherical volume over which the compensating charge is distributed uniformly. The electron may perform oscillations around its equilibrium position, thereby emitting light of its mechanical frequencies. In contrast to the subsequent mechanical models it was compatible with electrodynamics with regard to the emission of light. For several other reasons it turned out to be untenable. Spin had never been an issue for this model.

In 1911 Rutherford [2] positions the electron outside a nucleus with a compensating charge. It keeps itself away from the nucleus through its circular motion. The well-known and never refuted contradiction to electrodynamics: The orbiting charge continuously radiates and therefore shortly falls into the nucleus. – There were no considerations about spin for this model.

In 1913 Bohr [3] quantizes the Rutherford model with the help of additional quantum conditions and stabilizes it at this occasion through his radiation hypothesis: "On its quantized orbits the electron does not radiate". The contradiction to electrodynamics thereby is being cemented. A further contradiction to the Maxwell theory is added: Bohr finds himself compelled to decouple the radiation frequency from the mechanical orbital frequency of the electron. – Also in this model there was no spin.

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In 1915 Sommerfeld [4] extends the Bohr model to the general Kepler motion and by that completely covers the fine structure of the hydrogen spectrum. Still no spin, although Sommerfeld calculates already relativistically.

In 1925 Uhlenbeck and Goudsmit [5] completed the Sommerfeld model by the assumption that the orbiting electron possesses an intrinsic angular momentum because of a rotation around its own axis. With the spin quantum number $m = \pm(1/2)$ they assign to it the spectroscopically verifiable half-integer feature and ambiguity linked with the idea that two spin orientations are possible for the orbiting electron: in the direction of the orbital angular momentum or against it. This extension of the model did not emerge from calculational conclusions of the kind that the model itself produced spin, but from a concretely visible defect of the preceding model: It was only able to produce one half of the states that are necessary for a complete explanation of the hydrogen spectrum. Hence we have spin here for the first time, though not as an immanence of the model but as an additionally imposed requirement.

In 1925 Heisenberg [6] brought out hydrogen with the help of his quantum mechanics. It is the first model that quantizes itself, in contrast to the preceding quantized models that are charged with quantum conditions from outside. The model possesses no spin.

In 1926 Schrödinger [7] portrays hydrogen with the help of his one-component wave mechanics which he obtains by means of inserting the Hamilton refraction ("Hamilton analogy") into the classical light equation

$$\left(\Delta - \frac{N^2}{c^2}\frac{\partial^2}{\partial t^2}\right)\Psi = 0\,. \tag{1}$$

The model quantizes itself – as the one by Heisenberg. It possesses no spin.

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In 1927 Schrödinger [8] presents a one-component relativistic version of his 1926 model. It must be abandoned since it produces the wrong fine structure. Despite its relativistic calculation it has no spin.

In 1928 Dirac [9] depicts hydrogen through a four-component relativistic version of Schrödinger's 1926 model. It exhibits spin in accordance with Uhlenbeck and Goudsmit. Spin, though, is only visible in the frequency conditions of the model, not, however, in the four spinor components, in which it should articulate itself as half-integer spin factor. Dirac tries to cope with this grave defect of his model with a kind of emergency averaging procedure which he extracts, as far as possible, from the lower indices of the spherical harmonics of the four spinor components.

In 1990 the author [10] describes hydrogen with the help of electrodynamics. The (3+3)-component model shows a centrally transverse divector field which obeys the Pauli principle. It possesses half-integer spin. This appears not only – as with Dirac – in the frequency conditions but also articulates itself in a completely clear manner as half-integer spin factor

$$e^{i\phi/2}\, e^{-i\omega t} = e^{i(\phi/2-\omega t)} \tag{2}$$

in the electric and magnetic field components of the model.

2. Classical Light Equation and Newtonian Mechanics

In this section we convince ourselves, in preparation for the following, that the connection between Newtonian mechanics and ray optics, as being expressed by "Hamilton's analogy", can be confirmed completely and precisely. In particular we want to prove here that mechanics is contained within optics.

For this we start from the light equation (1), assume the usual harmonic time dependence

$$\Psi = \exp\left\{\frac{2\pi i}{h}\, U t\right\} \psi \qquad (U = h\nu) \tag{3}$$

and insert (by the "Hamilton analogy") the Hamilton refraction

$$N = c\,\frac{\sqrt{2m(U-\Phi)}}{U}. \tag{4}$$

Then the two Schrödinger equations

$$\left(\Delta + \frac{2m(U-\Phi)}{\hbar^2}\right)\Psi = 0 \tag{5}$$

and

$$\left[\Delta - 2m\left(\frac{\Phi}{\hbar^2} - \frac{i}{\hbar}\frac{\partial}{\partial t}\right)\right]\Psi = 0 \tag{6}$$

follow immediately. Now we consider, in accordance with Ehrenfest [11] but under a formal generalization of his procedure, the mean position coordinate

$$\bar{x}_1 = \int \Psi\Psi^* x_1\, \mathrm{d}V \tag{7}$$

and its two first time derivatives.

Concerning the first derivatives we deduce from (6) the relation

$$\frac{\partial}{\partial t}(\Psi\Psi^* x_1) = x_1\frac{\partial\Psi\,\Psi^*}{\partial t} = \frac{\hbar}{2mi}\, x_1(\Psi\Delta\Psi^* - \Psi^*\Delta\Psi) \tag{8}$$

and proceed with the identity

$$x_1(\Psi\Delta\Psi^* - \Psi^*\Delta\Psi) = \operatorname{div}(x_1\Psi\operatorname{grad}\Psi^* - x_1\Psi^*\operatorname{grad}\Psi) - \left(\Psi\frac{\partial\Psi^*}{\partial x_1} - \Psi^*\frac{\partial\Psi}{\partial x_1}\right), \tag{9}$$

which follows from the well known vector relation $\operatorname{div}(\Phi \boldsymbol{A}) = \boldsymbol{A}\operatorname{grad}\phi + \phi\operatorname{div}\boldsymbol{A}$. Thus (7) gives

$$\frac{\mathrm{d}x_1}{\mathrm{d}t} = \frac{i\hbar}{2m}\int_\infty \left(\Psi\frac{\partial\Psi^*}{\partial x_1} - \Psi^*\frac{\partial\Psi}{\partial x_1}\right)\mathrm{d}V, \tag{10}$$

because the integral over

$$-\frac{i\hbar}{2m}\operatorname{div}(x_1\Psi\operatorname{grad}\Psi^* - x_1\Psi^*\operatorname{grad}\Psi)$$

vanishes according to Gauss' theorem.

For the second derivative we again make use of (6), now to the effect

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\left(\Psi\frac{\partial\Psi^*}{\partial x_1} - \Psi^*\frac{\partial\Psi}{\partial x_1}\right) &= i\hbar\left(\Psi\frac{\partial^2\Psi^*}{\partial x_1\partial t} - \Psi^*\frac{\partial^2\Psi}{\partial x_1\partial t}\right) \\ &\quad + \Phi\left(\Psi\frac{\partial\Psi^*}{\partial x_1} + \Psi^*\frac{\partial\Psi}{\partial x_1}\right) \\ &\quad - \frac{\hbar^2}{2m}\left(\frac{\partial\Psi^*}{\partial x_1}\Delta\Psi + \frac{\partial\Psi}{\partial x_1}\Delta\Psi^*\right). \end{aligned} \tag{11}$$

In the first term on the right hand side, (6) is used once more yielding for the two first terms the equality

$$i\hbar\left(\Psi\frac{\partial^2\Psi^*}{\partial x_1\partial t} - \Psi^*\frac{\partial^2\Psi}{\partial x_1\partial t}\right) + \Phi\left(\Psi\frac{\partial\Psi^*}{\partial x_1} + \Psi^*\frac{\partial\Psi}{\partial x_1}\right) = \frac{\hbar^2}{2m}\left(\Psi\frac{\partial}{\partial x_1}\Delta\Psi^* + \Psi^*\frac{\partial}{\partial x_1}\Delta\Psi\right) - 2\Psi\Psi^*\frac{\partial\Phi}{\partial x_1}. \tag{12}$$

From (10) we thus derive the short equation

$$\frac{d^2 x_1}{dt^2} + \frac{1}{m} \int_{\infty} \Psi \Psi^* \frac{\partial \Phi}{\partial x_1} dV = 0 \tag{13}$$

because both parts of the integral over

$$\frac{\hbar^2}{2m} \left(\Psi \Delta \frac{\partial \Psi^*}{\partial x_1} - \frac{\partial \Psi^*}{\partial x_1} \Delta \Psi \right) + \frac{\hbar^2}{2m} \left(\Psi^* \Delta \frac{\partial \Psi}{\partial x_1} - \frac{\partial \Psi}{\partial x_1} \Delta \Psi^* \right)$$

vanish by Green's theorem.

Analogous relations are obtained for the second and third coordinate (x_2 and x_3). The vectorial combination then reads

$$m \frac{\partial^2 \bar{\boldsymbol{r}}}{dt^2} = \bar{\boldsymbol{K}} \quad \text{with} \quad \bar{\boldsymbol{K}} = -\operatorname{grad} \Phi . \tag{14}$$

For relatively short waves the mean force $\bar{\boldsymbol{K}}$ should be close to the value $\boldsymbol{K}(\bar{r})$ of the force field $-\operatorname{grad} \Phi$ at the mean position $\bar{r}$. Thus it has been shown that classical dynamics is approximately contained in the classical light equation (1) with the appropriate refractive index (2). In other words:

The intensity center of a light wave field moves in the short limit according to Newtonian mechanics.

This Theorem of Ehrenfest, however, only applies to classical optics. In electrodynamics the product of ε and μ appears instead of the Hamilton refraction N so that the theorem has to turn out more complicated.

3. Schrödinger's Hydrogen

Between (3) and (6) we have seen that the classical light theory (1), with the insertion of the Hamilton refraction (4), loses the signal velocity – and with that is relativistic character, or electrodynamic one, respectively. Schrödinger's wave mechanics, and especially Schrödinger's hydrogen, therefore are to be labeled as non-relativistic. This fact has found its confirming expression in the relation (20).

Since hydrogen can be described without problems with the help of electrodynamics [10] we negate in this section the Kopenhagen interpretation of the Schrödinger theory. Instead of this we envisage, in view of the electrodynamic hydrogen model, that in Schrödinger's hydrogen

1) the Schrödinger function is an optical scalar that represents any one of the six electromagnetic light components;
2) the inherent Kepler system is not made up by a proton and an electron, but by two "mass points" of equal weight, that is, the light-energy centers of the two partial fields Ψ_{Re} and Ψ_{Im};
3) the wave-particle scenery is reversed in going from the Copenhagen interpretation to the optical interpretation: In the Copenhagen interpretation the particle is the real entity and its wave the calculatory one, being the immaterial concept of a probability of presence. In the light interpretation, on the contrary, the field is the tangible, real entity, whereas the particle, represented by the center of the wave field, is the calculatory, unreal and immaterial one (as any center of gravity is).

The Hamilton analogy (4) now, of course, implies: A body moving in a potential behaves like a ray of light in a refraction or, applied to a Kepler system: Two mass points in mutual potential interaction behave like two light fields in mutual refraction.

Hence, for the hydrogen solution of (1), or (5) and (6), respectively, we shall have to deal with a two-body system, or a two-light-field system, where the two partial fields Ψ_{Re} and Ψ_{Im} mutually refract each other according to (4).

We take into account the inclusion of two masses, or wave fields, respectively, into the system (1), or (5) and (6), respectively, in the well-known manner, in that we introduce in (6) for m the reduced mass

$$m = \frac{m_{\text{Re}}\, m_{\text{Im}}}{m_{\text{Re}} + m_{\text{Im}}}, \tag{15}$$

where m_{Re} and m_{Im} denote the two masses of the system. In addition we require, in view of item 2 above, $m_{\text{Re}} = m_{\text{Im}}$, so that the relations

$$m_{\text{Re}} = m_{\text{Im}} = 2m \tag{16}$$

follow.

Finally we record that the Schrödinger function of hydrogen has the exponential form

$$\Psi^{\text{hyd}} = CRP\, e^{im\phi}\, e^{-i\omega t} = \Psi_{\text{Re}} + i\,\Psi_{\text{Im}} . \tag{17}$$

In order to point out illustrative and comprehensible features of the light-field system we investigate the motion of the intensity centers of the two light-fields. The barycenter of Ψ_{Re} and Ψ_{Im}, under the assumption that both functions are normalized, is given by

$$\bar{r}_{\mathrm{Re}} = \int_\infty \Psi_{\mathrm{Re}}^2\, \boldsymbol{r}\, \mathrm{d}V, \qquad \bar{r}_{\mathrm{Im}} = \int_\infty \Psi_{\mathrm{Im}}^2\, \boldsymbol{r}\, \mathrm{d}V, \tag{18}$$

$\boldsymbol{r}$ being the radius vector. The center of the total energy is obtained as their vector sum

$$\bar{r} = \int_\infty (\Psi_{\mathrm{Re}}^2 + \Psi_{\mathrm{Im}}^2)\, \boldsymbol{r}\, \mathrm{d}V = \int_\infty \Psi\Psi^*\, \boldsymbol{r}\, \mathrm{d}V. \tag{19}$$

Differentiating twice with respect to time yields

$$\ddot{\bar{r}} = \frac{\mathrm{d}^2}{\mathrm{d}t^2} \int_\infty \Psi\Psi^*\, \boldsymbol{r}\, \mathrm{d}V. \tag{20}$$

By (14) we obtain a connection with the potential gradient of classical mechanics:

$$\frac{\mathrm{d}^2\bar{r}}{\mathrm{d}t^2} = -\frac{1}{m}\, \overline{\mathrm{grad}\, \Phi}\,. \tag{21}$$

This tells us that the center of the total energy of the hydrogen atom approximately obeys Newtonian dynamics.

For the motion of the centers of gravity of the two partial fields Ψ_{Re} and Ψ_{Im}, according to (14), the analogous equations are

$$\frac{\mathrm{d}^2\bar{r}_{\mathrm{Re}}}{\mathrm{d}t^2} = -\frac{1}{m_{\mathrm{Re}}}\, \overline{\mathrm{grad}\, \Phi}, \qquad \frac{\mathrm{d}^2\bar{r}_{\mathrm{Im}}}{\mathrm{d}t^2} = -\frac{1}{m_{\mathrm{Im}}}\, \overline{\mathrm{grad}\, \Phi}\,. \tag{22}$$

The two centers of gravity of the partial fields, $\boldsymbol{r}_{\mathrm{Re}}$ and $\boldsymbol{r}_{\mathrm{Im}}$, therefore also move in some approximation according to Newtonian mechanics.

4. The Half-Integer Property

The general hydrogen solution of electrodynamics (59, 61) of [10] shows definitely that the spin of hydrogen appears conclusively as half-integer. The solution, however, does not state directly *why* spin adjusts itself exactly as half-integer.

Contrary to expectation the much simpler light-interpreted hydrogen, whose scalar Schrödinger function approximately represents just a single one of the six components of the electrodynamic model, allows an immediate view of the cause of the half-integer property of spin.

From mechanics it is well known that in a Kepler system with two equal masses the two masses lie opposite to each other at any time, antipodal with respect to the center symmetry. The same is true for the centers of energy of the two partial fields of hydrogen (17), because of (22). Looking, however, at the field energies of the two partial fields

$$\Psi_{\mathrm{Re}} = CRP \cos(m\phi - \omega t) \quad \text{and}$$
$$\Psi_{\mathrm{Im}} = CRP \sin(m\phi - \omega t)$$

we immediately find that the energies Ψ_{Re}^2 and Ψ_{Im}^2 do not place themselves antipodally, that is, turned by 180° with respect to each other. They rather come to lie just turned by 90° with respect to each other, thus, so to speak, "around the corner". To this a Kepler system would correspond where the two orbiting masses follow each other not with an angular distance of 180° but with one of 90° or 270°, respectively, an obvious impossibility.

If, however, we now introduce ad hoc into the two partial fields the electrodynamic spin-angular momentum factors, according to [14] (59, 60), by means of

$$e^{im\phi} = e^{i(m-1/2)\phi}\, e^{i\phi/2}\,, \tag{23}$$

we immediately get an antipodal energy distribution.

With (23) we have introduced spin ad hoc, which automatically arises in this form imperatively just in electrodynamic hydrogen. If the light-interpreted Schrödinger hydrogen (17) should bring about half-integer spin of itself, then one obviously has to enlarge the three usual wave mechanical "constraints" for the Schrödinger function, that is, finiteness, uniqueness and continuity, by the requirement of symmetry for the energy distribution. In addition, the condition of uniqueness has to be given up. – It will not be suitable to imagine the Schrödinger function, as modified through (23), as a dipole field because of the ambiguity of the right-hand side of (23). More successful should be, to view the modified Schrödinger field simply just as an approximation of any one of the six components that together constitute the electrodynamic hydrogen divector field.

The above considerations show that half-integer spin is a necessary prerequisite for a symmetric energy distribution in light-interpreted hydrogen (17).

5. Ether

It consists of,

1. according to the Presocratics:	water without waves,
2. according to Maxwell:	water and waves,
3. according to Einstein:	waves without water.

re 2: For Maxwell the existence of an ether was an absolutely necessary prerequsite for the substantiation of his theory [12]. The introduction of a displacement current, closing the circuit, brought about, after several inadequate preceding theories, a representation of the electromagnetic phenomena of such an adequacy to reality that until today just negligible defects of Maxwell's electrodynamics have become known. The heart of the now 100 years old theory is the displacement current, and it needs an ether in order to flow also in "empty space" ($\varepsilon = 1$, $\mu = 1$): no ether – no displacement current – no Maxwell theory. The elimination of ether by special relativity therefore has well to be met with great reserve. So far there exist many dealings with this matter but no productive way out. The problem has simmered along in the waiting loop for a long time. For, on the one hand, the primacy of the Maxwell theory is unquestionable in any relevant respect, the range of electrodynamics, as efficiency in cognition is concerned, is incomparably larger than the one of special relativity. On the other hand, of course, also the successful completion of the efforts of the relativists by a beautiful, exact and natural algorithm carries its weight not to be overlooked. With the elimination of ether, special relativity still has another serious problem on its back: Light has no carrier any more. An "independent physical entity" should the electromagnetic waves be, the relativists say [13]. Whereby the whole absurdity is transferred into the concept "entity".

But also ether has its problems. According to the requirements of the theories concerned it should pervade infinite space ubiquitously and stationary, penetrate all bodies, define an inertial system but not allow equality of status to other intertial systems., It may neither be subject to nor exert effects of gravity, offer no resistance to planetary motion, not take part in motions of bodies, not permit any longitudinal waves and be permeable only to transverse waves. And finally, the most essential and unrealizable pretension: because of the value of the speed of light it should, on the one hand, be highly rigid, or of extremely high tenacity, respectively, whereas on the other hand it should not impede the motion of planets. How should that be possible?

That seems easily possible if we – as in Sect. 3 – replace the Kopenhagen interpretation with the light interpretation and conceive the elementary particle as a standing light wave. From (14) then follows that the hydrogen atom, for instance, (or generally matter) moves through the tenaciously hard ether according to Newtonian mechanics, that it glides, for the case that no forces act, inertly ("straight and uniformly") through the solid ether with arbitrary velocity. With the disappearance of the discrepancy between the two requirements – extreme hardness and practical imperceptibility – within the light interpretation the classical world would look as follows: The universe consists of an infinitely extended, stationary, highly rigid, elastic, transparent and isotropic prime insulator (solid ether). In it the optics of (1) is valid. It generates with its perturbations all those plays of light that we, ourselves plays of light, see to proceed before our eyes as the colossal spectacle cosmos.

re 1: In some areas of the physical sciences it has become custom to look for a suitable counterpart for the basic structure of theories at the Greek ancestors – and to find them. So, for instance, the elementary particle physicists have their Demokritos and the quantum people struck gold twice with Heisenberg: They have Platon's theory of forms for the symmetries of quantum field theories, and for Heisenberg's uncertainty the Aristotelian potentia [14]. Altogether doubtlessly a rather meager output by the affinity potential that especially the Presocratics provide. Let us examine why.

The Greek natural philosophers, especially the School of Miletus (6th century BC) and the School of Elea (5th century BC), of course could not experiment yet, among others they did not possess any time-measuring yet. In contrast to today's physical research, where theoreticians try to incorporate experimental empirical results, the old natural philosophers had just pure cogitation on the one hand and just the immediate sensory perception on the other hand at their disposal for the formulation of their ideas. Pure cogitation gave them the central basic position of Greek natural phlosophy, the "being", an eternally invariable, omnipresent primary matter, whose existence could not be perceived by the senses but only be comprehended by reason. Apparent variety and change should, on the other hand, be simply things of the senses, the sensory perception could only grasp appearances.

An obvious assignment covering all requirements would be to view the being as the carrier of the matter waves of quantum field theory. But this is in conflict with its Kopenhagen interpretation. For probability waves, as pure abstractions, may have just an abstract carrier again, or no carrier at all. Here lies the ultimate reason why the Kopenhagen-interpreted quantum field theory can not offer to the Greek something analogous to the central position of the being.

Completely different is the situation for the case of the light interpretation. Here – without fundamental difficulty – the being can be put side by side with solid ether, and the things of the senses with its perturbations. – Infinitely extended, eternally invariable, omnipresent and not to be perceived by the senses, these signs of the being are identical with those fundamental properties we had to assign to solid ether, and that have been extended by the more recent natural sciences only by the respective experimental empirical results, that is, high rigidity, elasticity, transparency, isotropy and electrical non-conductivity. Exactly the same imaginable covering assignment exists between appearance, the things of the senses or illusions of the Greek and the optical perturbations of solid ether that generate light and matter, matter that moves according to Newtonian mechanics in view of (14) and, together with light, produces all those sensations that seemed just appearance to the Greek – in contrast to the being.

re 3: The elimination of ether by special relativity deprives, as already mentioned, electromagnetic waves of its carrier, – Einstein's ocean: an ocean of waves without water. The elimination of ether has been no act of wantonness, it suggests itself in virtue of the symmetry between two inertial systems of special relativity. The relativists, up to this day, have no acceptable answer to the notorious question: "What is waving in an electromagnetic wave?"

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